1	Large amplitude internal tides, solitary waves and
2	turbulence in the central Bay of Biscay
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8	Abstract
9	Microstructure and fine-scale measurements collected in the central Bay of Biscay
10	during MOUTON experiment are analyzed to investigate the dynamics of internal
11	waves and associated mixing. Large amplitude internal tides (ITs), that excite internal
12	solitary waves (ISWs) in the thermocline, are observed. ITs are dominated by modes
13	3 and 4, while ISWs projects on mode-1 that is trapped in the thermocline. Therein,
14	ITs generate a persistent narrow shear band, which is strongly correlated with the
15	enhanced dissipation rate in the thermocline. This strong dissipation rate is further
16	reinforced in presence of ISWs. Dissipation rates during the period without ISWs
17	largely agree with the MacKinnon-Gregg scaling proposed for internal wave fields
18	dominated by a low frequency mode. On the opposite, they show poor agreement with
19	the Gregg-Henyey parameterization valid for internal wave fields close to the GM
20	model. The agreement with the MacKinnon-Gregg scaling is consistent with the fact
21	that turbulent mixing is here driven by the low-frequency internal tidal shear.
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26 **1. Introduction**

It is well known that the Bay of Biscay (BB) is a marginal sea with large amplitude internal tides (ITs). Therein, ITs are mainly generated on the slope via interaction between the barotropic tide and the steep topography [e.g. *Pingree and New*, 1991; *Gerkema et al.*, 2004; *Pichon et al.*, 2011]. Once generated, they propagate both seaward and shoreward. Linear internal-wave theory and observations show that the seaward traveling ITs propagate along a beam or ray path with a slope to the horizontal given by:

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$$c=\pm[(\omega^2-f^2)/(N^2-\omega^2)]^{\frac{1}{2}},$$
 (1)

35 where ω is the semidiurnal frequency, f is the local inertial frequency and N is the 36 buoyancy frequency [Pingree and New, 1991]. The theory also predicts that the 37 offshore propagating beam first reflects at the bottom and next at the sea surface in 38 the central BB [Gerkema et al., 2004; Pichon et al., 2011] (Fig.1.a). Using 39 temperature and velocity records obtained from the central BB, New and Pingree 40 [1990] observed large internal interfacial tides and internal solitary waves (ISWs) in 41 the thermocline. These waves were as large as those observed at the shelf edge, and 42 they were thus conjectured to be locally aroused in the central BB, where internal 43 tidal beams hit the seasonal thermocline near the sea surface. This hypothesis was 44 confirmed by satellite imagery [New and Da Silva, 2002]. Theoretical framework for 45 this process was given by Gerkema (2001) (hereafter referred as to G01) who showed 46 that scattering of IT beams in the thermocline leads to the local generation of large 47 ITs and ISWs. This scenario was retrieved in other numerical models [Grisouard et al, 48 2011] and reproduced in laboratory experiments [Mercier et al, 2012].

While there are numerous insights on ITs and ISWs in the BB, there have neverbeen measurements in the BB allowing a fine characterization of ITs and ISWs.

Another aspect that has never been quantified is the turbulent mixing induced by these internal waves. In this paper, we report on joint fine-scale and microstructure measurements performed for the first time in the central BB. We first characterize vertical structures of ITs and ISWs and next dissipation rate (ε) of turbulent kinetic energy and turbulent mixing induced by ITs and ISWs. Eventually, fine-scale parameterizations are tested against microstructure measurements.

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58 **2. Data**

59 Data were collected at a location (45.75°N, 7.38°W) in the central BB during the 60 MOUTON experiment in summer 2008, where the water depth is about 4800 m (Fig. 61 1a). This location is about 200 km away from generation area of internal tidal beams on the northern slope of BB (Fig. 1a) [Pichon et al., 2011]. From Sept.18th, 62 63 01(h):10(min) until Sept. 19th, 04:00, combined CTD/LADCP yo-yoing was 64 performed. Sixteen profiles were collected with an effective depth range from 6 to 65 2663 m. From 04:50 to 21:20 on Sept.19th, two microstructure instruments, VMP (Vertical Microstructure Profiler) and SCAMP (Self-Contained Autonomous 66 67 Microstructure Profiler), were alternatively deployed. Dissipation rate estimates with 68 Scamp are based on Batchelor fitting of the temperature gradient spectrum measured 69 by high frequency temperature sensors [Ruddick et al., 2000], whereas VMP estimates 70 are based on measurements of microscale vertical shear. While 19 VMP profiles were 71 obtained from the surface to depths of 400-500 m, 15 Scamp profiles were collected 72 from the surface to depths of 70-110 m. Data above 10 m were removed due to the 73 contamination by the ship's wake. Current velocities were inferred from two 150-kHz 74 and 38-kHz broadband shipboard ADCPs. Measurement range of the former ADCP is 75 from 18 m to 162 m with a 2-min time-average interval and 4-m bin, while that of the 14 latter is from 55 m to 1231 m with the same time-average interval and 24-m bin. We 150 used here the 150-kHz ADCP data to compute 4-m shear (S_4) during microstructure 150 measurements.

79 CTD data were de-spiked and averaged into 1-m bins, while ε is computed from 80 VMP and Scamp data using 1m bins. Since Scamp was not equipped with 81 conductivity sensors, salinity during Scamp measurements was estimated by a linear 82 interpolation of VMP salinity data.

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84 **3. Results**

85 **3.1 Mean stratification and semidiurnal IT ray path**

86 The time-averaged CTD data were used to compute the buoyancy frequency N(z). 87 The climatological temperature and salinity data at the measurement location obtained 88 from the World Ocean Atlas 2009 were used to complete the profile below 2663 m 89 depth (Fig. 1b). The N(z) profile showing the largest value at 40-70 m (seasonal 90 thermocline) and a small peak near 800 m (permanent pycnocline) (Fig. 1b) is typical 91 for the central BB during summer [G01]. An internal tidal beam path is computed 92 using N(z) and a typical model topography in the BB. The surface reflection site of 93 internal tidal beams in the central BB is ~140 km away from their generation location 94 and ~60 km away from our observation site (Fig. 1a, embedded panel).

95 **3.2 Internal interfacial tides and ISWs**

Time series of temperature show large thermocline depressions with a semidiurnal period, so called as internal interfacial tides (Fig. 2a). Their amplitude from crest to trough is in the range 40-70 m. Note that the largest isotherm displacements are often observed below 1800 m (not shown). These large displacements are likely induced by the descending beam, which intersects the mooring position at large depth (see

101 inserted panel in Fig. 1a). During the second observation period, that of 102 VMP/SCAMP measurements, the isotherms slope in the thermocline depressions 103 become steeper, implying the enhancement of IT nonlinearity. As a result, high-104 frequency ISWs are generated in troughs and surface velocities are increased (Figs. 2b 105 and 3a). Note that the averaged time interval between two ISWs events is equal to 25-106 min, therefore ISWs could not be identified from temperature data due to their low 107 temporal resolution (about 20-min) but only from ADCP data.

108 We determine in the following the directions of propagation of ITs and ISWs as 109 well as their vertical structure. The large thermocline depressions often cause 110 southward velocity in the surface layer (Figs. 2c and 2d), implying a southward 111 propagation of ITs. These large southward interfacial tides likely result from the 112 scattering of the seaward traveling internal tidal beams generated on the northern slope [G01]. However, a more complex pattern can be observed on Sept.19th, 6h-12h. 113 114 During this period, two oppositely travelling troughs are observed in the large 115 thermocline depression: the first one generates southward surface velocity while the 116 second one induces northward surface velocity. The trough propagating northward 117 may be associated with internal tidal beams originating from the southern slope 118 [Pichon et al., 2011]. Since the velocity direction caused by ISWs is the same as that 119 of thermocline depressions (Fig. 2c), their propagating direction is consistent with that 120 of internal tides.

To investigate the vertical structure of ITs and ISWs, we have computed vertical modal functions of displacement $\Phi(z)$ and horizontal velocity $\Pi(z)$ at semidiurnal and 1/25 min⁻¹ frequencies following equations 6.10.2, 6.10.3 and 6.10.6 from *Gill* [1982]. Modal structures of waves with these two frequencies show large difference (Figs. 1c and 1d), as the high frequency is larger than *N* below the thermocline (see Fig. 1b). As

126 a result, the high-frequency modes are trapped in the thermocline (Figs. 1d and 2b). 127 To distinguish mode functions of high-frequency waves from that of low-frequency 128 waves, we refer to the former as trapped mode structures and to the latter as IT mode 129 thereafter. In order to infer IT mode structures, we compute time series of isopycnal 130 vertical displacement from CTD measurements. Then we extract the semidiurnal 131 amplitude for each depth using a harmonic analysis of the displacement time series. 132 The resulting normalized displacement profile at the semidiurnal frequency is plotted 133 in Figure 1c. The mode-3 structure roughly reflects depth variation of IT displacement 134 except for depths below 1800 m (Fig. 1g), where more modes may be present due to 135 the downward IT beam. A mode projection on IT displacements by the least square 136 method indicates that mode-3 (mode-4) explains about 61% (23%) of total potential 137 energy above the depth of 500 m. Accordingly, the sum of the projected mode-3 and 138 mode-4 reproduces well the variation of semidiurnal displacement above 500 m depth 139 (Fig. 1c). Meanwhile, measured velocity profiles when ISWs pass the fixed point are 140 well fitted by trapped mode-1 (Fig. 1i). Note that as vertical advection of velocity 141 signals by thermocline displacements is strong (see Fig. 2c), Eulerian depth of 142 velocity profiles in Figures 1h and 1i was transformed to semi-Lagrangian depth. 143 Interestingly, the vertical structures of IT mode-4 and trapped mode-1 are very similar 144 within the thermocline (Fig. 1e), which suggests that the observed ISWs are likely to result from mode-4 ITs. 145

146 **3.3 Microstructure measurements**

147 Time series of dissipation rates is shown in Figure 3b. The largest ε values often 148 occur near the surface, which may be directly associated with wind stirring in the 149 mixed layer. Enhanced ε values are also encountered in the thermocline with a mean ε 150 value of 2.3×10^{-8} Wkg⁻¹. Diapycnal diffusivity, K_{ρ} , was computed according to the 151 relationship established by Shih et al. (2005) who distinguished different regimes as a function of turbulent intensity: $K_{\rm p} = \lambda \varepsilon / N^2$, where $\lambda = 2[\varepsilon / (\nu N^2)]^{-1/2}$ (v is the molecular 152 viscosity) when $\varepsilon/(vN^2) > 100$ otherwise $\lambda = 0.2$. Enhanced K_0 , also occur in the 153 thermocline (Fig. 3c) with a mean value equal to $1.5 \times 10^{-5} \text{m}^2 \text{s}^{-1}$, slightly larger than 154 155 background values found in the open ocean thermocline. Below the thermocline, mixing becomes quite weak ($\varepsilon < 10^{-9} \text{ WKg}^{-1}$, $K_{\rho} < 10^{-5} \text{m}^2 \text{s}^{-1}$) except for a few patches. 156 157 The time-mean profiles along isotherms also display the above depth variations of ε 158 and K_{ρ} (inserted panels in Figs. 3b and 3c).

159 In the 4-m shear field, a narrow band of strong shear caused by ITs is clearly 160 identified within the thermocline with the strongest stratification (Fig. 3e). Because of 161 the large thermocline displacement caused by interfacial tides, the strong shear band is vertically elevated and depressed in a semidiurnal period. The time-averaged S^2 and 162 N^2 show the same variation with decreasing temperature/depth (inserted panel in Fig. 163 3d). However, S^2 is smaller than N^2 , implying that the Richardson number $Ri (=N^2/S^2)$ 164 165 is larger than 1 and that the large-scale waves are stable toward shear instability. 166 Since the strong stratification in the top of the thermocline (Fig. 3d) inhibits the 167 penetration of surface turbulence into the thermocline, the enhanced ε inside the 168 thermocline is likely to be associated with strong shears in the thermocline (Fig. 3e). 169 Below the thermocline, weaker ε is associated with smaller shear and stratification. In 170 addition, dissipation rates as well as K_{ρ} in the troughs are elevated relative to those in 171 the crests. This may be associated with disintegration of IT troughs into a group of 172 ISWs that further enhances turbulent mixing.

173 **3.4 Mixing Parameterization**

In order to investigate further the relationship between dissipation rates andshear/stratification, the observed dissipations rate were averaged into logarithmic bins

176 of N^2 and S^2 . Two parameterizations, MacKinnon-Gregg (M-G) and Gregg-Henyey 177 (G-H), are tested. The former was found to be relevant in situations where shear of 178 large-scale waves and energy of small-scale waves do not maintain a particular 179 relationship through the Garrett-Munk (GM) spectrum as illustrated for internal wave 180 mixing in the coastal ocean [*MacKinnon and Gregg*, 2003], while the latter applies 181 for internal wave fields with the GM spectral shape and is widely encountered in the 182 open ocean [*Gregg*, 1989]. These two parameterizations are given by

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$$\varepsilon_{GH} = 1.8 \times 10^{-6} f cosh^{-1} (N_0/f) (S/S_{GM})^4 (N/N_0)^2$$

184
$$S_{GM}^4 = 1.6 \times 10^{-10} (N/N_0)^2$$
 (2)

- 185 and
- 186 $\varepsilon_{MG} = \varepsilon_0 (N/N_0)(S/S_0),$ (3)

where $N_0 = S_0 = 0.0052 \text{ s}^{-1}$ and ε_0 is an adjustable constant. This constant was set as 187 2.2×10^{-9} Wkg⁻¹ using profiles without ISWs so that the mean ε_{MG} matches the 188 189 observed mean dissipation rate. Data within the mixed layer, namely those above the 190 thermocline (T>17.3°C), were removed for the comparison. Since ADCP data in the 191 lower laver have many gaps, only data above the depth of 110 m were used. 192 Meanwhile, 150-kHZ ADCP data were movingly averaged onto 30-min resolution 193 before computing shear, as we do not focus on high-frequency shear. Stratification data were smoothed by an 8-m Bartlett filter to agree with the spatial response of 194 195 ADCP. The dissipation rates were vertically averaged over 4-m. Since results 196 measured by different microstructure instruments may have bias, VMP and Scamp 197 data were calculated independently. The results are displayed in Figure 4.

During VMP measurements, high dissipation values are encountered in strong sheared and stratified regions (Fig. 4a). The distribution of the measured ε in the space of N^2 and S^2 is qualitatively more consistent with the M-G parameterization (Fig. 4b),

201 while it is evidently different from the G-H parameterization (Fig. 4c). The distribution pattern of the observed ε in the space of N^2 and S^2 during Scamp 202 203 measurement is also better reproduced by the M-G parameterization (Figs. 3d to 3f). However, in the regions of $7 \times 10^{-5} \text{s}^{-2} < N^2 < 3 \times 10^{-4} \text{s}^{-2}$, the observed ε are evidently 204 205 larger than those predicted by the parameterization. This may be associated with 206 nonlinear waves, whose mixing cannot be described by the above two 207 parameterizations [Mackinnon and Gregg, 2003]. Note that most of ISWs appear 208 during Scamp measurements (see Fig. 3). In addition, Ri is partly found to be 209 between 0.25 and 1, but no evident correlation between this parameter and dissipation 210 rate is identified.

To obtain the trend of dissipation rates versus S^2 and N^2 alone, the bin-sorted 211 dissipation rates are averaged along N^2 and S^2 , respectively (Figs. 4g-4j). The two 212 213 parameterizations show large difference in the trend of dissipation rates versus shear (Figs. 4g and 4i), while the M-G model during VMP measurements successfully 214 reproduces the slope of the observed dissipation rates with increasing S^2 . During 215 Scamp measurements, the M-G model also captures the essential relationship of 216 dissipation rate versus shear, although the dissipation rates are larger for $6 \times 10^{-6} s^{-2}$ 217 $< S^2 < 1 \times 10^{-4} s^{-2}$ due to the effect of nonlinear waves. Regarding the relationship 218 219 between dissipation rate and stratification (Figs. 4h and 4j), both models reflect the 220 main variation of dissipations rates with stratification.

221

222 **4. Discussion**

In the previous section, we showed that the elevated turbulent mixing in the thermocline mainly resulted from strong low-frequency (IT) shears. There large ITs do not directly cause mixing because of their stability (Ri>0.25), but they are likely to 226 directly promote energy transfers toward smaller scale waves that ultimately become 227 unstable leading to mixing. These conditions are consistent with the simple 228 assumption behind the M-G scaling. As a result, we find that the observed dissipation 229 rate is largely reproduced by the M-G model. The simple assumption behind the M-G 230 scaling is that the large-scale shear providing the environment for scattering and 231 breaking of smaller waves results from a few low-frequency waves (ITs or near-232 inertial waves) rather than from a steady state background GM spectrum on which the 233 G-H scaling applies. This assumption is generally fulfilled in shallow costal seas such 234 as New England Shelf [MacKinnon and Gregg, 2003, 2005] and the Baltic Sea [van 235 der Lee and Umlauf, 2011], where dissipations were also well parameterized by the 236 M-G model. Therefore, the M-G scaling is mostly known as a shallow water 237 parameterization. However, our observations in the deep sea that fulfill the M-G 238 assumptions, with an internal wave field dominated by a few vertical modes at the 239 semidiurnal frequency, show that the M-G scaling has a wider range of applications 240 than coastal seas.

241 The large ITs that we observe in the thermocline propagate southward and mostly 242 project on mode-3 and mode-4. It is quite likely that they result from scattering of IT 243 beams generated in the northern slope of BB when beams propagate upward from the 244 deep water and hit the thermocline following the scenario described by G01. In this 245 course, nonlinear steepening of ITs further excites ISWs which are as well observed at 246 the mooring location. Since the frequency of ISWs exceeds the buoyancy frequency 247 except in the thermocline, these waves are trapped in this region where they may 248 further increase local mixing. However because of the limited number of 249 microstructure profiles during ISW's events and the long time interval between two

- adjacent profiles, it was difficult to give evidence of the impact of ISWs on mixing.
- 251 Additional data are required for this purpose.
- 252
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301 Captions

302 Figure 1: (a) Map showing bathymetry in the BB and the observation site (the black 303 point). The slash line indicates the possible propagation path of the seaward traveling 304 internal tidal beams generated on the northern slope. Their characteristic path is given 305 in the inserted panel (blue line with arrows). The red curve is typical model 306 topography in the BB along the cross-slope direction. The vertical line indicates the 307 observation location. (b) Profile of N(z): The vertical line indicates a frequency of $1/25 \text{ min}^{-1}$. Vertical structures of displacement $\Phi(z)$ for the mode-1 (blue), mode-2 308 309 (green), mode-3 (red) and mode-4 (pink) waves with (c) semidiurnal and (d) 1/25 min⁻ ¹ frequencies. In (c), the black solid line is the measured IT displacement profile 310 around 3h/19th (black solid line). The dashed line is sum of projected mode-3 and 311 312 mode-4. (e) Observed velocity profiles (red and blue lines) during two ISWs (marked 313 by arrows in Figure 3a) in a semi-Lagrangian frame and velocity profiles of IT mode-314 4 (black dashed line) and trapped mode-1 (black solid line). Note that both 315 displacements and velocities have been normalized.

Figure 2: (a) Temperature. White vertical lines indicate the times of CTD profiles. (b)
Current Velocity amplitude. Time series of meridional velocities obtained from (c)
150-kHZ and (d) 38-kHZ ADCPs. Data in (d) have been movingly averaged to 10min resolution. Two black lines in (c) are isotherms of 17.3°C and 12.9°C, indicating
depths of the thermocline, similar for Figures 3b to 3e (white lines in Figures 3d and
3e).

Figure 3: (a) Velocity amplitudes during VMP/Scamp observation. The arrows indicate ISWs plotted in Figure 2b. (b) Dissipation rate. (c) Diacypnal diffusivity. Isotherms whose interval is 0.5° C (white lines) are over-plotted on (b) and (c). (d) N^2 .

- 325 (e) $(S_4)^2$. A 4-m running average is used for (b) and (c). Data in (e) have 30-min 326 resolution. The embodied panels in (b), (c) and (d) is time-averaged ε , K_{ρ} and N^2 (blue) 327 and $(S_4)^2$ (red) along the isotherms
- **Figure 4:** Distribution of dissipation rates in bins of N^2 and S^2 from observations (a
- and d) and the M-G (b and e) and G-H (c and f) parameterizations. The averaged
- dissipation rate in bins of S^2 (g and i) and N^2 (h and j), respectively. (a), (b), (c), (g)
- and (h) for the observation period of VMP. (d), (e), (f), (i) and (j) for the observation
- 332 period of Scamp. The red, blue and green curves in the latter 4 panels are the results
- 333 of the observation, M-G model and G-H model, respectively. The grey shading
- indicates 95% confidence interval. The oblique solid and dashed lines in (a) to (f) are
- the boundaries of Ri=0.25 and Ri=1.



Figure 1



Figure 2



Figure 3



Figure 4